IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Comment on 'Geometric phases for mixed states during cyclic evolutions'

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2004 J. Phys. A: Math. Gen. 37 7393

(http://iopscience.iop.org/0305-4470/37/29/N01)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.91
The article was downloaded on 02/06/2010 at 18:24

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. **37** (2004) 7393–7394 PII: S0305-4470(04)80047-7

COMMENT

Comment on 'Geometric phases for mixed states during cyclic evolutions'

Erik Sjöqvist

Department of Quantum Chemistry, Uppsala University, Box 518, Se-751 20 Uppsala, Sweden

E-mail: erik.sjoqvist@kvac.uu.se

Received 30 April 2004
Published 7 July 2004
Online at stacks.iop.org/JPhysA/37/7393
doi:10.1088/0305-4470/37/29/N01

Abstract
It is shown that a recently suggested concept of mixed state geometric phase in cyclic evolutions (Fu L-B and Chen J-L 2004 *J. Phys. A: Math. Gen.* **37** 3699) is gauge dependent.

PACS numbers: 03.65.Vf, 03.67.Lx

Fu and Chen [1] have proposed a concept of geometric phase for mixed quantal states in cyclic unitary evolution. In the present paper, we demonstrate that this phase is gauge dependent and thus not a property of the path in state space.

Consider the unitary path

$$\mathcal{C} : t \in [0, \tau] \to \rho(t) = U(t)\rho(0)U^{\dagger}(t) \tag{1}$$

of the density operator $\rho(0)$. The evolution is cyclic iff $[\rho(0), U(\tau)] = 0$. In such a case, $\mathcal{C}$ is closed and the mixed state phase ϕ_g proposed in [1] takes the form

$$\phi_g = \sum_k w_k \phi_g^k \tag{2}$$

where w_k are the time-independent eigenvalues of the density operator and

$$\phi_g^k = \mathrm{i} \int_0^{\tau} \langle \psi_k(t) | \, \mathrm{d}\psi_k(t) \rangle \tag{3}$$

are the cyclic pure state geometric phases [2] associated with the corresponding eigenvectors $|\psi_k(t)\rangle$ chosen to be periodic, i.e., $|\psi_k(\tau)\rangle = |\psi_k(0)\rangle, \forall k$.

Now, the periodicity of the eigenvectors is preserved under the transformation $|\psi_k(t)\rangle \to |\tilde{\psi}_k(t)\rangle = \mathrm{e}^{-\mathrm{i}\alpha_k(t)}|\psi_k(t)\rangle$ if $\alpha_k(\tau) - \alpha_k(0) = 2\pi n_k$, n_k integer. It follows from equation (3) that

$$\phi_g^k \to \tilde{\phi}_g^k = \mathrm{i} \int_0^{\tau} \langle \tilde{\psi}_k(t) | \, \mathrm{d}\tilde{\psi}_k(t) \rangle = \phi_g^k + 2\pi n_k \tag{4}$$

i.e., $\mathrm{e}^{\mathrm{i}\tilde{\phi}_g^k} = \mathrm{e}^{\mathrm{i}\phi_g^k}$. On the other hand, by inserting equation (4) into equation (3), we obtain

$$\phi_g \to \tilde{\phi}_g = \sum_k w_k \tilde{\phi}_g^k = \phi_g + 2\pi \sum_k w_k n_k \tag{5}$$

where the additional term on the right-hand side is in general not an integer multiple of 2π, i.e., $\mathrm{e}^{\mathrm{i}\tilde{\phi}_g} \neq \mathrm{e}^{\mathrm{i}\phi_g}$.

Note that the path $\mathcal{C}$ is invariant under the gauge transformation $U(t) \to U(t)V(t)$ if $[\rho(0), V(t)] = 0, \forall t \in [0, \tau]$. Explicitly, we may take

$$V(t) = \sum_k \mathrm{e}^{-\mathrm{i}\alpha k(t)} |\psi_k(0)\rangle\langle\psi_k(0)| \tag{6}$$

with $\alpha_k(t)$ given above. Due to its dependence upon $V(t)$ via the integers n_k, it follows that ϕ_g is not a property of the closed path $\mathcal{C}$. In other words, the phase concept proposed in [1] is not gauge invariant.

A gauge invariant mixed state geometric phase has been proposed in [3]. In the cyclic case, this phase γ is determined by

$$\mathcal{V}\,\mathrm{e}^{\mathrm{i}\gamma} = \sum_k w_k\, \mathrm{e}^{\mathrm{i}\phi_k^g} \tag{7}$$

and is independent of n_k as each phase factor in the sum of the right-hand side transforms invariantly. This phase may be tested interferometrically as an incoherent average of pure state interference profiles upon elimination of all the pure state dynamical phases in one of the arms. For cyclic evolution, such an analysis yields the intensity

$$\mathcal{I} \propto \sum_k w_k \left| \mathrm{e}^{\mathrm{i}\chi} |\psi_k(0)\rangle + \mathrm{e}^{\mathrm{i}\phi_g^k} |\psi_k(0)\rangle \right|^2 \propto 1 + \mathcal{V}\cos[\chi - \gamma] \tag{8}$$

where χ is a variable $U(1)$ shift applied to the other arm. γ has recently been measured using nuclear magnetic resonance technique [4].

In conclusion, we have demonstrated that a quantal phase concept recently proposed in [1] is gauge dependent and therefore not a property of the path in state space. Thus, this phase is neither experimentally testable, nor does it qualify as a geometric phase for mixed states during cyclic evolutions.

References

[1] Fu L-B and Chen J-L 2004 *J. Phys. A: Math. Gen.* **37** 3699

[2] Aharonov Y and Anandan J 1987 *Phys. Rev. Lett.* **58** 1593

[3] Sjöqvist E, Pati A K, Ekert A, Anandan J S, Ericsson M, Oi D K L and Vedral V 2000 *Phys. Rev. Lett.* **85** 2845

[4] Du J F, Zou P, Shi M, Kwek L C, Pan J-W, Oh C H, Ekert A, Oi D K L and Ericsson M 2003 *Phys. Rev. Lett.* **91** 100403